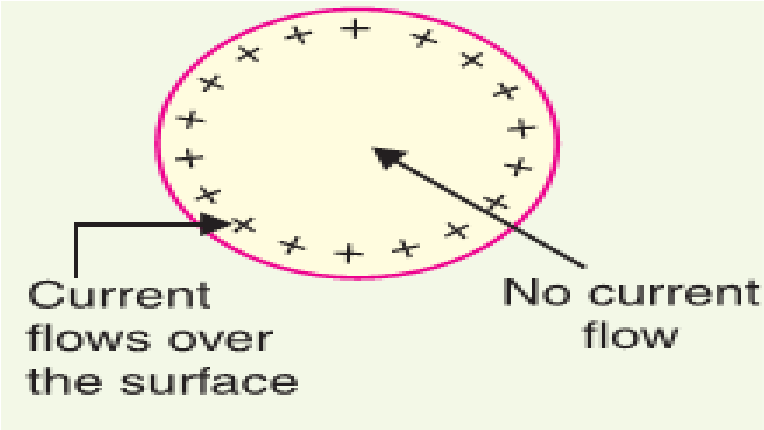
Skin Effect

- The tendency of alternating current to concentrate near the surface of a conductor is known as skin effects.

  - ✓ When a conductor is carrying steady direct current (D.C), this current is uniformly distributed over the whole X-section of the conductor. (**For a (D.C)current, inductance is zero and hence the current distributes uniformly over the entire X-section of the conductor**)
  - ✓ The alternating current (A.C)flowing through the conductor does not distribute uniformly, rather it has the tendency to focus near the surface of the conductor as shown in Fig. below. This is known as skin effect.



- By reason of skin effect, the effective area of cross-section of the conductor through which current flows is reduced. As a result, the resistance of the conductor is slightly increased when carrying an alternating current.

The cause of skin effect can be easily explained.

- A solid conductor may be thought to be consisting of a large number of strands, each carrying a small part of the current.
- The inductance of each strand will vary according to its position.
- Thus, the strands near the centre are surrounded by a **greater magnetic flux** and hence have **larger inductance** than that near the surface.
- The high reactance of **inner strands** causes the alternating current to flow near the surface of conductor.
- This crowding of current near the conductor surface is the **skin effect**.

- ✓ It may be noted that skin effect is negligible when the supply frequency is low (< 50 Hz) and conductor diameter is small (< 1cm).

Ex.1

Determine the resistance of a 10-km-long solid cylindrical aluminum conductor with a diameter of 250 mils, at (*a*) 20°C and (*b*) 120°C.

To find the cross-sectional area of the conductor, we note that

$$250 \text{ mils} = 0.25 \text{ in} = 0.635 \text{ cm}$$

so

$$A = \frac{\pi}{4}(0.635)^2 = 0.317 \text{ cm}^2$$

Also, from Table 3-1, $\rho = 2.83\ \mu\Omega \cdot \text{cm}$ and $\alpha = 0.0039°\text{C}^{-1}$ at 20°C.
(*a*) At 20°C, (*3.1*) yields

$$R_{20} = \rho\frac{l}{A} = 2.83 \times 10^{-8} \times \frac{10 \times 10^3}{0.317 \times 10^{-4}} = 8.93\ \Omega$$

(*b*) At 120°C, (*3.2*) yields

$$R_{120} = R_{20}[1 + \alpha(120 - 20)] = 8.93(1 + 0.0039 \times 100) = 12.41\ \Omega$$

Ex2

A transmission-line cable consists of 19 strands of identical copper conductors, each 1.5 mm in diameter. The length of the cable is 2 km but, because of the twist of the strands, the actual length of each conductor is increased by 5 percent. What is the resistance of the cable? Take the resistivity of copper to be $1.72 \times 10^{-8}\ \Omega \cdot \text{m}$.

Allowing for twist, we find that $l = (1.05)(2000) = 2100$ m. The cross-sectional area of all 19 strands is $19(\pi/4)(1.5 \times 10^{-3})^2 = 33.576 \times 10^{-6}\ \text{m}^2$. Then, from (*3.1*),

$$R = \frac{\rho l}{A} = \frac{1.72 \times 10^{-8} \times 2100}{33.576 \times 10^{-6}} = 1.076\ \Omega$$

EX3

A sample of copper wire has a resistance of 50 Ω at 10°C. What must be the maximum operating temperature of the wire if its resistance is to increase by at most 10 percent? Take the temperature coefficient at 10°C to be $\alpha = 0.00409°\text{C}^{-1}$

Here we have $R_1 = 50\ \Omega$ and $R_2 = 50 + 0.1 \times 50 = 55\ \Omega$. Also, $T_1 = 10°\text{C}$, and we require T_2. From (*3.2*) we obtain

$$55 = 50[1 + 0.00409(T_2 - 10)] \qquad \text{or} \qquad T_2 = 34.45°\text{C}$$

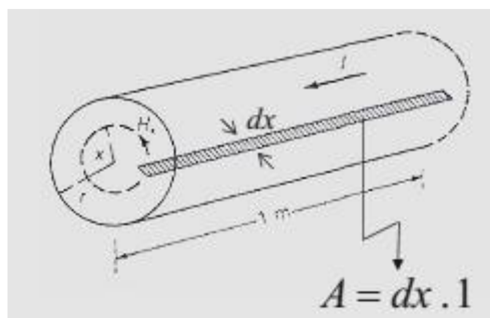
Inductance (Flux Linkages)

The inductance of a circuit is defined as the flux linkages per unit current. Therefore, in order to find the inductance of a circuit, the determination of flux linkages is of primary importance.

1. **Flux linkages due to a single current carrying conductor.** Consider a long straight cylindrical conductor of radius r meters and carrying a current I amperes (r.m.s.) as shown in Fig. 9.4 (i). This current will set up magnetic field. The magnetic lines of force will exist inside the conductor as well as outside the conductor. Both these fluxes will contribute to the inductance of the conductor.

   (i) Flux linkages due to internal flux. Refer to Fig. 9.4 (ii) where the X-section of the conductor is shown magnified for clarity. The magnetic field intensity at a point x metres from the centre is given by;

Fig. 9.4



$$mmf = NI$$
$$mmf = Hl$$
$$H = \frac{mmf}{l} = \frac{NI}{l} \quad (\frac{AT}{m})$$

Where
H – field intensity .
l - length of path of flux .
$l = 2\pi x$
N – No. of turns of conductor { for our analysis N=1 (one conductor)}
$\therefore Hl = I$

Ampere's law : [The *mmf* around any closed path ($l = 2\pi x$) equals the current enclosed by the path (Ix)]
Or
$2\pi xHx = Ix$

Where
Hx - is tangent of curve, it is constant.
Ix - is the portion of total current enclosed by the contour.
If current density in the conductor is uniform;

$$I_x = \frac{\pi x^2}{\pi r^2} I$$

I - total current in the conductor .

$$\therefore \quad H_x = \frac{I_x}{2\pi x} = \frac{\pi x^2}{\pi r^2 . 2\pi x} I = \frac{x}{2\pi r^2} I \qquad AT/m$$

We know : $B = \mu H$

Where B – flux density .
μ - permeability of conductor .

And , $B = \mu_r \mu_0 H$

Where μ_r - relative permeability
μ_0 - permeability of free space
$\mu_0 = 4\pi \times 10^{-7} \qquad henrys/meter$

$$\therefore \quad B_x = \frac{\mu x}{2\pi r^2} I \qquad Wb/m^2$$

Also , we know : $B = \dfrac{\Phi}{A}$

Where Φ - flux (Weber)
A – area (m^2)

The differential flux dΦ per – unit length of conductor in the cross - hatched rectangle of width dx shown in fig. above , is :

$$\Phi = BA \quad ; \quad d\Phi = B_x . dA$$

$$\therefore\ d\Phi = \frac{\mu x I}{2\pi r^2}\, dA = \frac{\mu x I}{2\pi r^2}\, dx.1 \quad Wb/m$$

Where

*d*Φ - flux enclosed in element of thickness *dx* per meter (axial) length of conductor .

So the flux linkage per meter length of conductor:

$$d\psi = \frac{\pi x^2}{\pi r^2}\, d\Phi \ = \ \frac{\mu x^3 I}{2\pi r^4}\, dx \qquad Wb.T/m$$

Integrating above eq. from x = 0 to x = r determine the total flux linkages **ψint** . inside the conductor :

$$\psi_{\text{int.}} = \int_0^r \frac{\mu x^3}{2\pi r^4}\, I\, dx = \frac{\mu I}{2\pi r^4}\int_0^r x^3\, dx$$

$$= \frac{\mu I}{2\pi r^4}\left[\frac{x^4}{4}\right]_0^r \ = \ \frac{\mu I}{8\pi} \qquad Wb.T/m$$

For nonmagnetic conductor , $\mu_r = 1$;

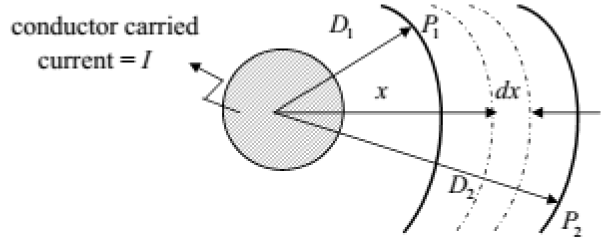
therefore , $\mu = \mu_0 = 4\pi \times 10^{-7}$

$$\therefore\ \psi_{\text{int.}} = \frac{\mu_0 I}{8\pi} = \frac{4\pi \times 10^{-7}}{8\pi} = \frac{1}{2} I \times 10^{-7} \qquad Wb.T/m$$

The internal inductance ***Lint*** . per – unit length of conductor due to this flux linkage is then :

$$\boxed{L_{\text{int.}} = \frac{\psi_{\text{int.}}}{I} = \frac{10^{-7}}{2} \qquad henrys/meter}$$

(ii)Flux linkages due to external flux. Now let us calculate the flux linkages of the conductor due to external flux. The external flux extends from the surface of the conductor to infinity. Fig. 9.5, the field intensity at a distance x -metres (from centre) outside the conductor is given by;



$$mmf = NI = Hl$$
$$l = 2\pi x \quad ; \quad N = 1$$
N – No. of turn

$$\therefore \quad 2\pi x H_x = I_x = I$$

$$H_x = \frac{I}{2\pi x} \qquad AT/m$$

$$B = \mu H = \frac{\Phi}{A}$$

$$B_x = \frac{\mu I}{2\pi x} \qquad Wb/m^2$$

$$\Phi = BA \quad ; \quad d\Phi = B_x.dA$$

The flux links with current I in the thickness dx :

$$d\Phi = \frac{\mu I}{2\pi x} dx \qquad Wb/m$$

The flux linkage per meter :

$d\psi = d\Phi$ (flux external to the conductor links all the current in the conductor).

The flux linkages between $P1$ and $P2$:

$$\psi_{12} = \int_{D_1}^{D_2} \frac{\mu I}{2\pi x} dx = \frac{\mu I}{2\pi} \ln \frac{D_2}{D_1} \qquad Wb.T/m$$

For nonmagnetic conductor, $\mu r = 1$;

therefore , $\mu = \mu_0 = 4\pi \times 10^{-7}$

$$\psi_{12} = 2 \times 10^{-7} I \ln \frac{D_2}{D_1} \qquad Wb.T/m$$

$$\boxed{\therefore \quad L_{12} = \frac{\psi_{12}}{I} = 2 \times 10^{-7} \ln \frac{D_2}{D_1} \quad henrys/m}$$

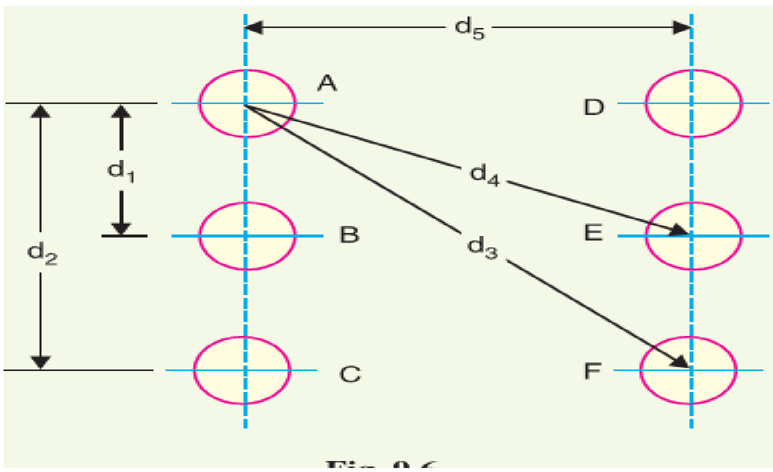
- Inductance $L12$, this only for flux linkage of an isolated conductor which lie between the points $P1$ and $P2$ distant $D1$ and $D2$ respectively from the center of the conductor as shown in fig. above.

2- Flux linkages in parallel current carrying conductors.

- Consider a group of parallel conductors A, B, C… etc, as shown in the Fig. below.
- Each of these conductors carry current I_A, I_B, I_C etc.

- Consider the flux linkages with conductor say A. There will be flux linkages with conductor A due to its own current.
- There will be flux linkages with this conductor due to the mutual inductance effects of IB, IC, ID etc.

We shall now determine the total flux linkages with conductor A.



Flux linkages with conductor A due to its own current,

$$= \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \int_r^\infty \frac{dx}{x}\right] \quad ...(i)$$

Flux linkages with conductor *A* due to current I_B

$$= \frac{\mu_0 I_B}{2\pi}\int_{d_1}^\infty \frac{dx}{x} \quad ...(ii)$$

Flux linkages with conductor *A* due to current I_C

$$= \frac{\mu_0 I_C}{2\pi}\int_{d_2}^\infty \frac{dx}{x} \quad ...(iii)$$

∴ Total flux linkages with conductor *A*

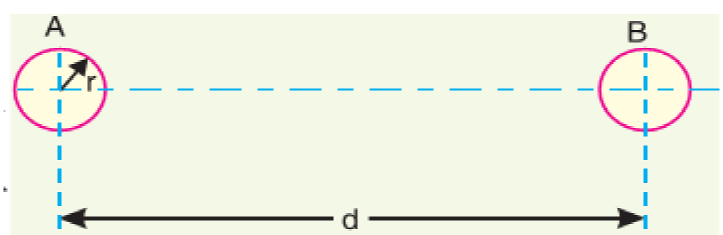
$$= (i) + (ii) + (iii) + ...$$

$$= \frac{\mu_0 I_A}{2\pi}\left(\frac{1}{4} + \int_r^\infty \frac{dx}{x}\right) + \frac{\mu_0 I_B}{2\pi}\int_{d_1}^\infty \frac{dx}{x} + \frac{\mu_0 I_C}{2\pi}\int_{d_2}^\infty \frac{dx}{x} + ...$$

- ❖ Similarly, flux linkages with other conductors can be determined. The above relation provides the basis for evaluating inductance of any circuit.

Inductance of a Single Phase Two-wire Line

- ❖ Consider a single phase overhead line consisting of two parallel conductors A and B spaced d meters apart as shown in Fig. below. Conductors A and B carry the same amount of current (i.e. IA = IB), but in the opposite direction because one forms the return circuit of the other.
  ∴ IA + I B = 0
- ❖ To find the inductance of conductor A (or conductor B), we shall have to consider the flux linkages with it.
- ❖ There will be flux linkages with conductor A due to its own current IA and
- ❖ Also due to the mutual inductance effect of current IB in the conductor B.



For external flux only

$$L_{1ext} = 2\times10^{-7}\ln\frac{D}{r_1} \quad henrys/m$$

For internal flux only

$$L_{1int.} = \frac{10^{-7}}{2} \qquad henrys/meter$$

- ✓ **Therefore , total inductance of the circuit due to current in conductor 1 only is :**

$$L_1 = L_{int} + L_{ext}$$

$$L_1 = \frac{1}{2}10^{-7} + 2\times10^{-7}\ln\frac{D}{r_1}$$

$$= 2\times10^{-7}\left(\frac{1}{4} + \ln\frac{D}{r_1}\right) = 2\times10^{-7}\left(\ln e^{1/4} + \ln\frac{D}{r_1}\right)$$

$$= 2\times10^{-7}\ln\left(\frac{D}{r_1}\times e^{1/4}\right) = 2\times10^{-7}\ln\left(\frac{D}{e^{-1/4}\times r_1}\right)$$

But , $e^{-1/4} = 0.7788$

$$L_1 = 2\times10^{-7}\ln\left(\frac{D}{0.7788\, r_1}\right)$$

$$\boxed{L_1 = 2\times10^{-7}\ln\left(\frac{D}{r_1'}\right) \qquad henrys/m}$$

- ✓ Where , $r1' = .07788\ r1$

  ***r*1′ is called Geometric mean radius (G.M.R)**

$r1' = 0.7788$ times the radius of conductor.

Also, the inductance due to current in conductor No.2 :

$$L_2 = 2\times10^{-7}\ln\left(\frac{D}{r_2'}\right) \qquad henrys/m$$

Total inductance for the complete circuit :

$$L = L_1 + L_2 = 2\times10^{-7}\left(\ln\frac{D}{r_1'} + \ln\frac{D}{r_2'}\right)$$

$$= 2\times10^{-7}\ln\left(\frac{D^2}{r_1' r_2'}\right)$$

$$= 4\times10^{-7}\frac{1}{2}\ln\left(\frac{D^2}{r_1' r_2'}\right) = 4\times10^{-7}\ln\left(\frac{D^2}{r_1' r_2'}\right)^{1/2}$$

Yln(a/b) = ln(a/b)^Y

$$= 4\times10^{-7}\ln\left(\frac{D}{\sqrt{r_1' r_2'}}\right)$$

If the radius of the two conductor is same , i.e :

$r_1' = r_2' = r'$, therefore $\sqrt{r_1' r_2'} = r'$

$$\therefore \quad L = 4\times10^{-7}\ln\left(\frac{D}{r'}\right) \qquad henrys/m \quad (H/m)$$

And ,

$$L = 0.4\ln\left(\frac{D}{r'}\right) \qquad mH/Km$$

Reactance X =2πfL

- **Sometimes this inductance is called inductance per loop length; it is double the inductance per conductor in a single phase line.**